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# Spatial and Temporal Variability of Soil Nitrate and Corn Yield: Multifractal Analysis

Bahman Eghball,\* James S. Schepers, Mehrdad Negahban, and Michael R. Schlemmer

## ABSTRACT

High levels of residual soil  $\text{NO}_3\text{-N}$  can contaminate ground water by leaching through the soil. Our objective was to reduce the level and spatial variability of residual soil  $\text{NO}_3\text{-N}$  while maintaining optimum corn (*Zea mays* L.) production by variable rate N fertilizer application. The experiment was located on a 60-ha sprinkler-irrigated corn field in central Nebraska and included four N management practices: uniform rate, variable rate (VRAT), variable rate at 75% of recommended amount (VRAT @ 75%), and variable rate plus 10% (VRAT + 10%). VRAT @ 75% decreased the amount of residual  $\text{NO}_3\text{-N}$  in the soil while maintaining similar grain yield to the other treatments, indicating over-application of N with treatments receiving the recommended rate. Increasing the recommended rate by 10% (VRAT + 10%) did not increase corn yield or residual soil  $\text{NO}_3\text{-N}$ . Based on multifractal spectrum, no consistent pattern of spatial variability of soil  $\text{NO}_3\text{-N}$  was observed for each treatment across years. Spatial variability in corn grain yield was much lower than that for soil  $\text{NO}_3\text{-N}$ , indicating noneffectiveness of using soil  $\text{NO}_3\text{-N}$  spatial distribution for variable rate N application unless some areas in the field are severely N deficient. Variable rate N application did not reduce variability of residual soil  $\text{NO}_3\text{-N}$  or corn grain yield as compared with uniform N. Multifractal analysis quantitatively characterized the extent and pattern of spatial and temporal variability in corn grain yield and residual soil nitrate.

RECENT DEVELOPMENTS in agricultural technology have made site-specific fertilizer application a reality. Variable rate (site-specific) N application should provide the plant with the appropriate amount of N while reducing the quantity and variability of residual soil  $\text{NO}_3\text{-N}$  after harvest. One may also expect to find a more homogeneous yield response across the field following adoption of variable rate N application. By reducing variability and quantity of residual soil  $\text{NO}_3\text{-N}$ , its leaching and subsequent ground water contamination potential should be reduced. Eghball et al. (1999) found that the extent of variability in residual soil  $\text{NO}_3\text{-N}$  was significantly reduced following adoption of variable rate N application in a continuous corn system under gravity irrigation. The residual soil  $\text{NO}_3\text{-N}$  to a depth of 0.9 m was high (avg.  $6.8 \text{ mg kg}^{-1}$ , max. 12.0 and min. 2.4) across the field before initiation of variable rate N application. After 1-yr variable rate N application, average residual soil  $\text{NO}_3\text{-N}$  was  $5.0 \text{ mg kg}^{-1}$  with a maximum of 7.9 and a minimum of 3.7. In another study where residual soil  $\text{NO}_3\text{-N}$  was low (avg.  $4.0 \text{ mg kg}^{-1}$ , max. 7.8 and min. 1.5), variable rate N application did not significantly reduce residual soil  $\text{NO}_3\text{-N}$  variability (Eghball et al.,

1997). Ferguson et al. (2002) found reduction in soil nitrate concentration due to variable rate fertilizer N application in only 3 out of 12 site-years as compared with uniform N application. Machado et al. (2000) indicated that management zones for variable rate fertilizer and water applications should be based on information about soil elevation, texture, and soil nitrate. Spatial dependence of soil  $\text{NO}_3\text{-N}$  was found to be time dependent in irrigated salad crops (Bruckler et al., 1997).

Fractal analysis can provide insight into the spatial or temporal variability of crop or soil parameters. Fractal analysis has been shown to be useful in a variety of scientific disciplines. The use of fractals for numerical analysis of soil and plant parameters is still a relatively new technique. It has been used for characterizing soil structure (Eghball et al., 1993b; Perfect and Blevins, 1997), soil chemical and physical parameters (Burrough, 1983), root morphology (Eghball et al., 1993a), temporal yield variations (Eghball and Power, 1995; Eghball and Varvel, 1997), and spatial variability of soil and crop yield (Eghball et al., 1997, 1999). Fractal analysis was found to be useful in characterizing soil and plant parameters that was not possible or very difficult to do before. Fractal dimension (D) of a curve can have a value between 1 and 2, giving a quantitative indication of the function's shape or roughness.

Multifractal analysis has been proposed for determination of spatial variability of soil parameters (Folorunso et al., 1994; Kravchenko et al., 1999, 2000). Multifractal parameters were found to reflect many of the major aspects of variability in soil properties, provided a unique quantitative characterization of the data spatial distribution, and multifractal parameters were useful in choosing an appropriate interpolation procedure for mapping soil properties (Kravchenko et al., 1999). Multifractal analysis was used to characterize particle-size distribution of soils with wide range of particle sizes (Posadas et al., 2001). A single fractal dimension might not be sufficient to characterize soil spatial variability because of the heterogeneous nature of soil parameters. A set of fractal dimensions, called a *multifractal spectrum*, is referred to as *multifractal analysis* (Frisch and Parisi, 1985). Multifractal analysis needs to be evaluated to determine its usefulness in comparing spatial variability of soils treated with different treatments. The objective of this study was to characterize and compare spatial and temporal variability of residual soil  $\text{NO}_3\text{-N}$  and corn grain yield in a variable rate N study using multifractal analysis.

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**Abbreviations:** *adiff*, the distance between minimum and maximum *a* values of each multifractal spectrum; CEC, cation exchange capacity; VRAT, variable rate; VRAT @ 75%, variable rate at 75% of the recommended amount; VRAT + 10%, variable rate of the recommended amount plus 10%.

## MATERIALS AND METHODS

### Field Treatments

An experiment was conducted from 1994 to 1997 on a 60-ha center-pivot irrigated corn field located in central Nebraska. The soil types within the field included 40% Blendon loam, 0 to 1% slope (coarse loamy, mixed, superactive, mesic Pachic Haplustolls), 20% Blendon loam, 1 to 3% slope, and 40% Hord silt loam, 0 to 1% slope (fine-silty, mixed, superactive, mesic Cumulic Haplustolls). Growing season rainfalls (1 May–31 October) were 418, 448, 528, and 458 mm while average temperature [(maximum + minimum) ÷ 2] were 19.3, 18.1, 18.3, and 18.6°C for 1994, 1995, 1996, and 1997, respectively. Four N management practices were applied to 32-row wide strips (24.4 m) that ran the entire length of the field (780 m). The management practices used were arranged in a randomized complete block design with five replications. Soil samples were collected in the spring to determine NO<sub>3</sub>-N level in the soil, which was utilized to calculate the N application rate for each treatment. The treatments were (i) fixed uniform N rate based on a strip average of soil NO<sub>3</sub>-N and organic matter obtained from grid sampling, (ii) variable rate N applied at 100% of recommended rate determined at each of the grid sample points based on the soil NO<sub>3</sub>-N and organic matter found at that point, (iii) variable rate N applied at 75% of recommended rate with the remainder being applied through fertigation if needed based on chlorophyll meter readings (N was applied only once in 1996 at a rate of 34 kg ha<sup>-1</sup> using a high clearance applicator), and (iv) variable rate N plus an additional 10% of the recommended rate. All practices used the University of Nebraska N recommendation equation for corn (Hergert et al., 1995). The NO<sub>3</sub>-N in irrigation water (26 mg L<sup>-1</sup>) for an expected application of 23 cm water yr<sup>-1</sup> was considered when recommended amount of N was calculated. Nitrogen fertilizer was applied as a sidedress application of anhydrous NH<sub>3</sub> at growth stages V6 to V9. Anhydrous NH<sub>3</sub> was applied with a toolbar-mounted coulter/knife injection unit placed into the furrow midway between plant rows. Nitrogen application rate was set either manually for the uniform N treatment, or adjusted according to field position by a SoilTeq<sup>1</sup> Falcon controller for the VRAT treatments. Nitrogen application maps were developed using SoilTeq SGIS software with grid soil sample data. Treatments were applied to the same strips each year. Nitrogen application rates are given in Table 1. The soil nitrate and grain yield maps were generated by kriging the data using GS<sup>+</sup> (Gamma Design Software, Plainwell, MI).

Soil samples were collected on a 12.2 by 24.4 m staggered grid to a depth of 0.9 m. Nitrate content of the soil samples was determined by extracting the soil with 2 M KCl and then using a Lachat system (Zellweger Analytics, Milwaukee, WI). A yield monitor linked to the global positioning system (John Deere Green Star) was used for the 1995, 1996, and 1997 harvest. Yield was obtained at each location that soil was sampled by averaging the yield monitor data from a 12.2-m-side rectangle centered around the soil sampling point.

Statistical analysis of the residual NO<sub>3</sub>-N and grain yield was performed using PROC MIXED (Littell et al., 1996) and adjusting the means for the spatial variability of the data. In this analysis, the means for each treatment is adjusted based

<sup>1</sup> Mention or use of a product does not imply endorsement by the USDA-ARS or the University of Nebraska.

**Table 1. Mean N fertilizer rates for uniform (UM) and variable (VRAT) treatments.**

Treatment	1994	1995	1996	1997	Avg.
	kg ha <sup>-1</sup>				
UM	106	131	123	173	133
VRAT	105	127	120	167	130
VRAT/UM	0.99	0.97	0.98	0.97	0.98

on spatial variability of the data in the entire field as determined by geostatistics' parameters (nugget, sill, and range). For details see pages 303 to 330 of Littell et al. (1996). Multifractal analysis was performed using a Fortran computer program written by the authors.

### Multifractal Analysis

The field was a square of 780 by 780 m divided into five segments (replications). Each segment contained randomly assigned treatment strips. As a result, the distance between strips of each treatment varied throughout the field. As shown in Fig. 1a for the uniform N treatment, each sampling point was assigned to a rectangular grid. When analyzing a treatment, it was assumed that the entire field received that treatment. This is similar to grid sampling of certain strips in a field to characterize spatial distribution of certain soil property in the entire field. The sides of these rectangular grids were defined by the midpoints between consecutive sampling points, or were defined by the field boundaries. When a sampling point was missing, depending on whether there were readings on both sides of the point or not, either the reading on one side, the other side, or the average of both sides was assigned to the missing value. When consecutive readings were missing, this process was started from the closest nonzero reading and repeated until every rectangle had a number assigned to it. This defined the data grid.

For the multifractal analysis, a set of cells was constructed. The field was subdivided into 4, 9, 16, and 25 square (Fig. 1b, 1c, 1d for 4, 9, 16, respectively) cells. As a result, the cell hypotenuse size  $\delta$  was 552, 367, 276, and 221 m, respectively. The hypotenuse size was used instead of the side so that the method would be applicable whether square or rectangular cells were used. For each treatment, actual data from the five strips (replications) were used for the multifractal analysis, with the missing data filled as described above.

The multifractal analysis method was as follows: For each treatment, the mass probability function ( $u_i$ ) for multifractal cell  $i$  (Fig. 1b, 1c, 1d) with hypotenuse size  $\delta$  was evaluated as

$$\mu_i(\delta) = \frac{M_i}{M} \quad [1]$$

where

$$M_i = \int_{A_i} F dA \quad [2]$$

and

$$M = \sum_{i=1}^n M_i \quad [3]$$

where  $F$  is the initial yield or NO<sub>3</sub>-N value in each data grid (assuming similar value as the sampling point for the entire area of each grid), the integral is a double integral over the area  $A_i$  of multifractal cell  $i$ . The integral is a double integral over the area  $A_i$  of multifractal cell  $i$  (data value was multiplied by its sampling grid area and summed for cell  $i$ ), and  $n$  represents the total number of cells of size  $\delta$ .

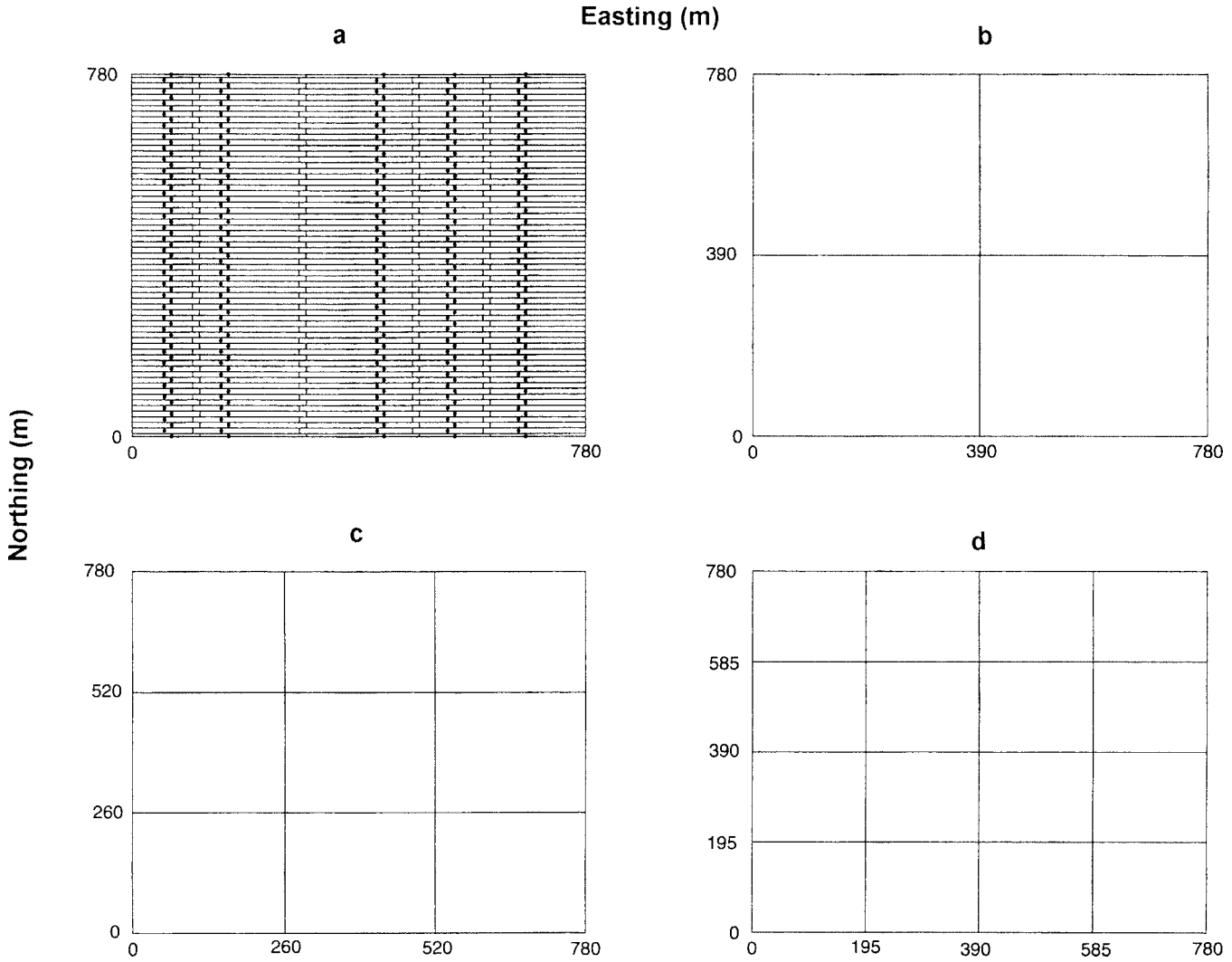


Fig. 1. The grid arrangements for the multifractal analysis. (a) is the sampling grids of the soil  $\text{NO}_3\text{-N}$  and grain yield for the uniform N treatment (sampling point is indicated by a dot); (b), (c), and (d) are different multifractal cell sizes.

The distribution of the probability mass function was then analyzed for multifractality using the method of moments (Evertsz and Mandelbrot, 1992). Briefly, a partition function was determined as follows:

$$\chi_q(\delta) = \sum_{i=1}^n \mu_i^q(\delta) \quad [4]$$

where  $q$  is a real number ranging from  $-\infty$  to  $\infty$ . For multifractally distributed measures, the partition function scales with the grid size as

$$\chi_q(\delta) \propto \delta^{\tau(q)} \quad [5]$$

where  $\tau(q)$  is the mass exponent of order  $q$ . The mass exponent for each  $q$  value can be obtained by plotting  $\log \chi_q(\delta)$  vs.  $\log \delta$ . If the probability function  $\mu_i(\delta)$  in the neighborhood of the grid scales with the grid size as  $\mu_i(\delta) \propto \delta^\alpha$ , then as  $\delta \rightarrow 0$ , the singularity exponent  $\alpha$  is a scaling property specific to the cell size. Parameter  $\alpha$  is also called a *local fractal dimension* or a *singularity index*. The local fractal dimension can be determined by Legendre transformation of the  $\tau(q)$  curve (Evertsz and Mandelbrot, 1992) as

$$\alpha(q) = d\tau(q)/dq \quad [6]$$

and be used to determine the multifractal spectrum (discussed

next). The number of cells of size  $\delta$  with the same  $\alpha$ ,  $N_\alpha(\delta)$ , is related to the cell size as  $N_\alpha(\delta) \propto \delta^{-f(\alpha)}$ , where  $f(\alpha)$  is a scaling exponent of the cells with common  $\alpha$ . Parameter  $f(\alpha)$  can be calculated as

$$f[\alpha(q)] = q \alpha(q) - \tau(q) \quad [7]$$

A plot of  $f(\alpha)$  vs.  $\alpha$  is called a *multifractal spectrum*. Multifractal spectrum quantitatively characterizes variability of soil or crop parameters with asymmetry to the right and left indicating domination of small and large values, respectively. The width of the multifractal spectrum (*adiff*) indicates overall variability similar to the nugget effects in geostatistics. For each treatment we calculated multifractal spectrum with  $q$  values ranging from  $-10$  to  $+10$  in increments of  $0.2$ . The  $f(\alpha)$  spectrum is related to the commonly used generalized multifractal dimension as

$$D(q) = \tau(q)/(q - 1) \quad [8]$$

The fractal dimension at  $q = 0$  is the box-counting dimension of the geometric support of the measure being studied, which in our case was Euclidian dimension of a plane (i.e., 2), information fractal dimension,  $D_1$ , is obtained at  $q = 1$  using the l'Hôpital's rule, and the correlation fractal dimension,  $D_2$ , is obtained at  $q = 2$ . The lower  $D_1$  or  $D_2$  values indicate domina-



**Table 2.** Mean comparisons of corn grain yield for four treatments in 3 yr at Shelton, NE.

N treatment	1995	1996	1997
	Mg ha <sup>-1</sup>		
Uniform N	10.18a*	12.18a	11.48ab
Variable N	10.15ab	12.09a	11.60a
Var. N @ 75%	9.98c	12.19a	11.43b
Var. N + 10%	10.06bc	12.14a	11.24c
CV, %	5.7	5.3	7.7

\* Within each column, the values are significantly different at the 0.05 probability level if different letters appear.

tion of long-range variation while higher values indicate domination of short-range variation.

## RESULTS AND DISCUSSION

### Corn Grain Yield and Residual Soil Nitrate

The total N application rates were not different between uniform and variable rate N application methods as the ratio of VRAT/uniform N was >0.97 (Table 1). The software used to control fertilizer application when this study was conducted was not capable of recording actual N rate applied; therefore, mean N rates for the VRAT treatments are reported. Variable rate N application did not result in less N application. Corn grain yields were influenced by the N application methods in 1995 and 1997 (Table 2). The coefficients of variation for grain yields were small, indicating low overall variability of grain yield in each year. Variable rate @ 75% resulted in less corn yield than VRAT in 1995, while both VRAT @ 75% and VRAT + 10% resulted in less yield than VRAT in 1997 (Table 2). However, because of low variability, the magnitude of the difference was small (max. of 360 kg ha<sup>-1</sup>). Adding 10% more N to the recommended rate seems to have negatively influenced corn grain yield in 1997. However, reducing the recommended amount by 25% did not result in less yield than other treatments in 2 out of 3 yr, indicating over-application of N using the recommended rate and/or more water applied than the expected 23 cm (26 mg L<sup>-1</sup> NO<sub>3</sub>-N in irrigation water).

The VRAT @ 75% treatment resulted in less residual soil NO<sub>3</sub>-N than other treatments indicated the effects of lowering the N application rate on soil residual NO<sub>3</sub>-N (Table 3). Increasing the application rate by 10% in the VRAT + 10% treatment did not result in greater residual soil NO<sub>3</sub>-N than the uniform or variable rate N treatments (Table 3). The CV values were high for residual soil NO<sub>3</sub>-N, indicating large overall variability. The VRAT and Uniform N application methods

resulted in similar residual soil NO<sub>3</sub>-N in all years except in 1996, where VRAT resulted in 0.6 mg kg<sup>-1</sup> less soil NO<sub>3</sub>-N than uniform N application (Table 3). Variable rate N application was not effective in reducing residual soil NO<sub>3</sub>-N as compared with uniform application unless the application rate was reduced by 25%. Eghball et al. (1999) also found no difference between variable rate and uniform N applications for spatial variability in residual soil NO<sub>3</sub>-N. The soil-based N recommendation used in this study seems to have underestimated N mineralization from soil. Ferguson et al. (2002) also reported that Nebraska N recommendation equation may not be appropriate for variable rate N application. In irrigated systems, a plant-based N management system utilizing corn canopy remote sensing and fertigation may be a viable alternative to soil-based recommendations because of the uncertainty about N mineralization in the soil. Another alternative might be to apply a reduced rate (perhaps 50%) of N fertilizer based on soil testing results and follow up with variable rate N application as an in-season treatment as needed based on remote sensing or crop stress data (Varvel et al., 1997).

### Multifractal Analysis

Coefficient of variation provides an indication of variability in the overall data. Multifractal analysis was performed to provide indication of the pattern and nature of spatial and temporal variability in the soil residual NO<sub>3</sub>-N and corn grain yield data. Advantage of multifractal analysis over traditional statistics and geostatistics were tested by Kravchenko et al. (1999) where they exchanged 40 random subsamples of cation exchange capacity (CEC) values in a field with the highest CEC values. In this case, statistical properties of the data set remained intact and only spatial structure of the data distribution was modified. Relocation did not change the statistical properties or the variogram, but multifractal spectra differentiated between the original and the modified data set.

Monofractal analysis performed on the residual soil NO<sub>3</sub>-N and corn grain yield data indicated significant differences among replications of each treatment for fractal dimensions, pointing out the heterogeneity of variability (data not shown). For a discussion on the use of monofractal analysis for characterizing and comparing spatial and temporal variability see Eghball et al. (1999). Because of heterogeneity of variability, multifractal analysis was performed on the combined data from all five replications of each treatment.

### Residual Soil Nitrate

If  $D(q)$  values decrease for increasing parameter  $q \geq 0$ , then the measure is called *multifractal* (Peitgen et al., 1992, p. 737). Regression lines of  $\log \chi_q(\delta)$  vs.  $\log \delta$  at different  $q$  values were linear for all treatments in all 4 yr with  $R^2$  values > 0.99, indicating excellent fit of the models used. The  $D(q)$  values were decreasing for increasing  $q$  values, indicating the multifractal nature of spatial variability of soil NO<sub>3</sub>-N (Fig. 2). The greatest decrease of  $D(q)$  with increasing  $q$  was observed for

**Table 3.** Mean comparison of residual soil NO<sub>3</sub>-N for four treatments in 4 yr at Shelton, NE.

N treatment	1994	1995	1996	1997
	mg kg <sup>-1</sup>			
Uniform N	5.35a*	6.03a	11.91a	3.46a
Variable N	5.22a	6.19ab	11.30b	3.50a
Var. N @ 75%	4.69b	5.86b	9.05d	3.06b
Var. N + 10%	5.15a	6.34a	9.97c	3.75a
CV, %	46	58	36	56

\* Within each column, the values are significantly different at the 0.05 probability level if different letters appear.

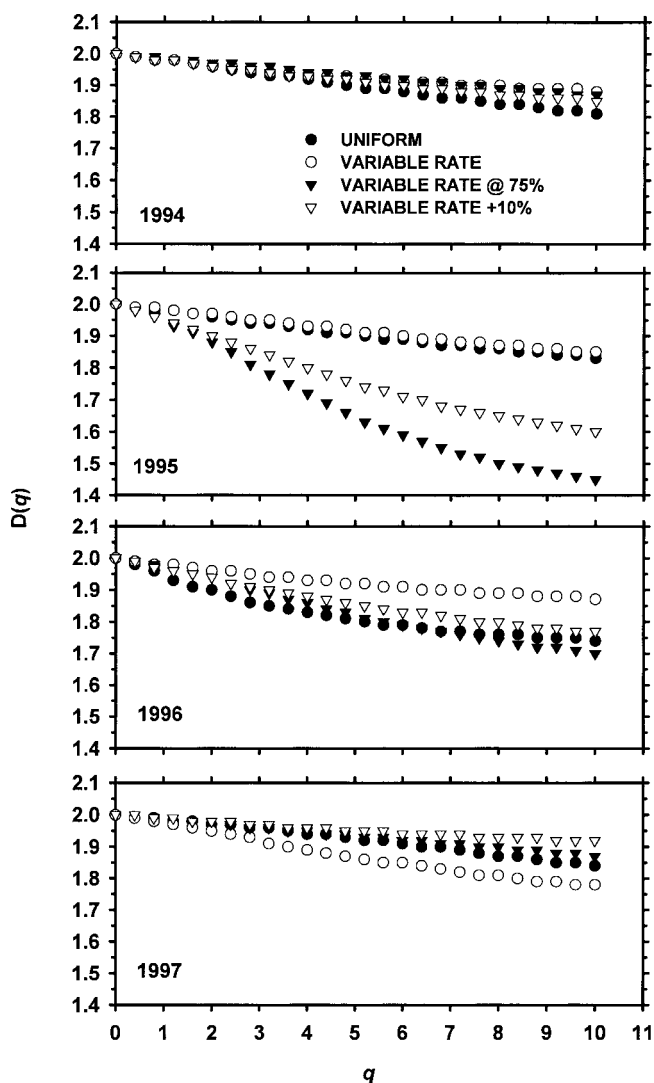


Fig. 2. Residual soil nitrate  $D(q)$  values as a function of increasing  $q$  values for four N treatments in 4 yr.

VRAT @ 75% and VRAT + 10% treatments in 1995 (Fig. 2). Multifractal spectrums of residual soil  $\text{NO}_3\text{-N}$  are presented in Fig. 3. Asymmetry toward the left from  $a = 2$  indicates domination of large or presence of extremely large values in the spatial variability pattern while asymmetry to the right indicates domination of small or presence of extremely small values. In 1994 when initial soil measurements were made before treatment applications, all four treatments had similar variability patterns with VRAT treatment skewed toward the right of  $a = 2$  (Fig. 3). In 1995, there were significant differences among the treatments for spatial variability patterns with VRAT @ 75% and VRAT + 10% skewed to the left, indicating domination of large values in spatial distribution of soil  $\text{NO}_3\text{-N}$  for these two treatments (Fig. 3). Soil  $\text{NO}_3\text{-N}$  maps for the four treatments in 1995 are shown in Fig. 4. The distribution pattern of soil  $\text{NO}_3\text{-N}$  for the VAR @ 75% and VAR + 10 treatments indicated large areas of low soil  $\text{NO}_3\text{-N}$  values ( $2.5\text{--}4.5 \text{ mg kg}^{-1}$ ) while VRAT and Uniform treatments had a more uniform distribution of the low and

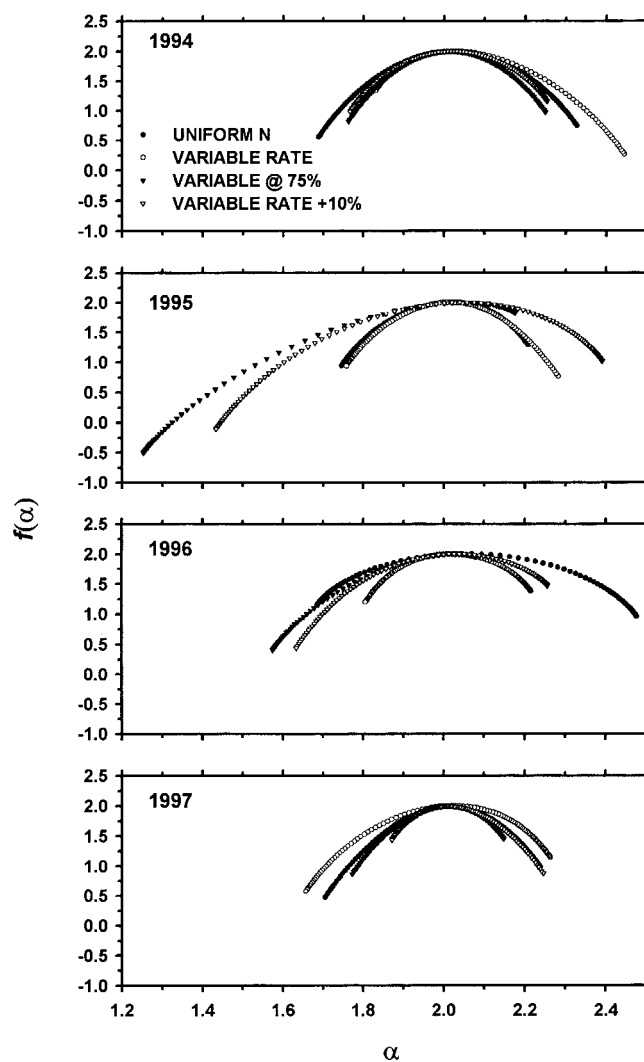


Fig. 3. Multifractal spectrums for residual soil  $\text{NO}_3\text{-N}$  for four N treatments in 4 yr.

medium values (Fig. 4). The very high soil  $\text{NO}_3\text{-N}$  values were concentrated in the right corners of the field for VRAT @ 75% and VRAT + 10% treatments. These high values skewed the multifractal spectrum to the left for  $a < 2$ . In 1996 and 1997, soil  $\text{NO}_3\text{-N}$  data also showed differences in spatial variability patterns among treatments, but the patterns were not consistent among years.

The  $D_1$  and  $D_2$  values for residual soil  $\text{NO}_3\text{-N}$  are given in Table 4. Smaller  $D_1$  or  $D_2$  values indicate domination of long-range variation while higher values indicate domination of short-range variability in the spatial pattern. The differences among treatments for  $D_1$  and  $D_2$  of residual soil  $\text{NO}_3\text{-N}$  were significant for all years except 1994, when the samples were collected before initiation of the treatments. The  $D_1$  and  $D_2$  values were smaller for the VRAT @ 75% and VRAT + 10% treatments in 1995, indicating domination of long-range variability (Fig. 4). This substantiated the patterns in the multifractal spectrums for these treatments (Fig. 3).

The distance between minimum and maximum  $a$  values for each multifractal spectrum ( $\text{adiff}$ ) is an important indicator of variability distribution. Kravchenko et al.

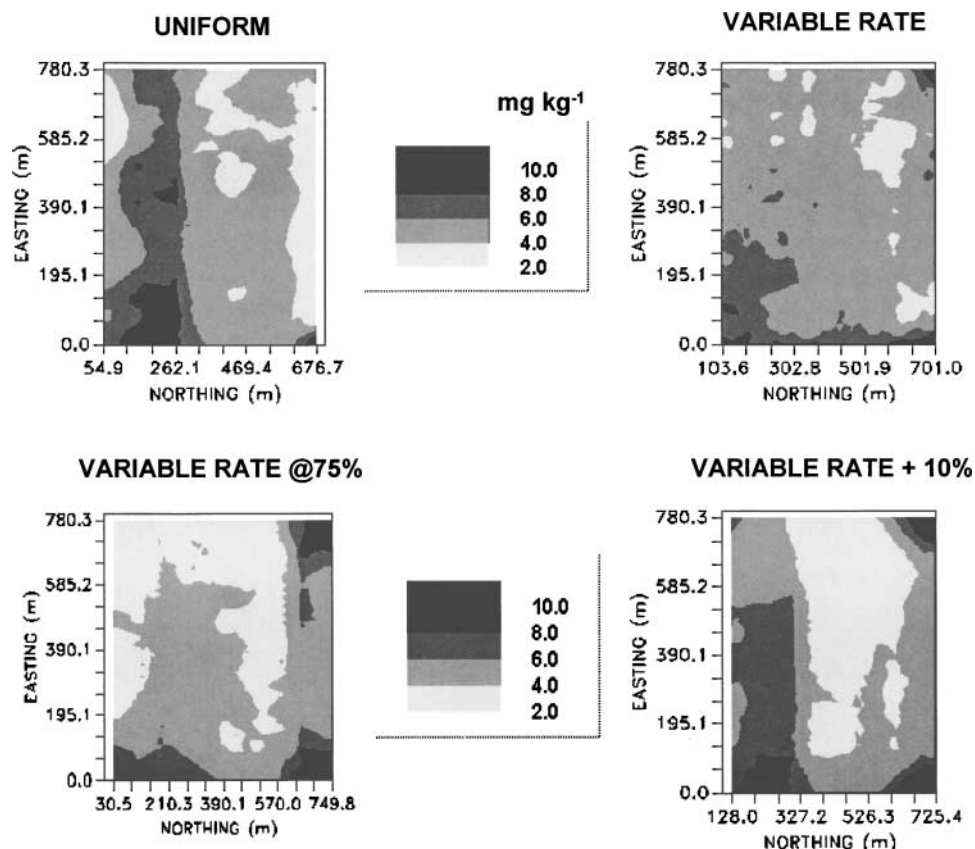


Fig. 4. Residual soil  $\text{NO}_3\text{-N}$  distribution for four treatments in 1995. Each map was generated using the data from the strips of that treatment.

(1999) found that *adiff* was highly correlated with the nugget effects when performing geostatistics on the same data. Analysis of variance of *adiff* values (using year as replication) indicated no significant effect of year ( $P = 0.35$ ) or treatment ( $P = 0.98$ ) main effects across years on *adiff*. The distribution of variability was not significantly different among treatments across years, pointing out the inconsistency of temporal spatial variability for each treatment. However, the multifractal spectrums were significantly different among treatments in each year, indicating different variability pattern for each (Table 5). The differences among treatments became more pronounced with years following initiation of the treatments.

### Corn Grain Yield

Regression lines of  $\log \chi_q(\delta)$  vs.  $\log \delta$  were linear for all treatments in all 3 yr with  $R^2$  values  $> 0.99$ . However,

the  $D(q)$  values remained relatively constant for increasing  $q$  values (Fig. 5). The multifractal spectrum indicated the least variability in 1995 and slight increase in asymmetry to the right in 1996 and 1997 for the treatments (Fig. 6). It seems that the spatial variability was increasingly dominated by the average values of corn grain yield each year. Corn grain yield maps for the four treatments in 1997 are shown in Fig. 7. As indicated in the maps, the spatial variability was not much different among treatments and no strong domination of small or large values was apparent. The *adiff* values also were not different among treatments (Table 5). It seems that the strong spatial variability of soil  $\text{NO}_3\text{-N}$  did not influence spatial variability patterns in corn grain yield, indicating adjustment by corn plants for spatial variability in N availability. Adequate N was available throughout the field, even though soil  $\text{NO}_3\text{-N}$  distribution showed

Table 4. Information ( $D_1$ ) and correlation ( $D_2$ ) fractal dimensions of residual soil nitrate for four treatments across 4 yr.

N treatment	1994		1995		1996		1997	
	$D_1$	$D_2$	$D_1$	$D_2$	$D_1$	$D_2$	$D_1$	$D_2$
Uniform N	1.98	1.96	1.98	1.96	1.94	1.90	1.99	1.97
Variable N	1.98	1.96	1.98	1.97	1.98	1.96	1.97	1.95
Var. N @ 75%	1.99	1.97	1.95	1.88	1.97	1.94	1.99	1.98
Var. N + 10%	1.98	1.96	1.95	1.90	1.97	1.94	1.99	1.98
SD	0.01	0.01	0.02	0.03	0.02	0.02	0.01	0.01
PR > T†	NS	NS	0.03	0.04	0.03	0.02	0.01	NS

† The probability level tests the hypothesis that the mean difference among treatments is equal to zero; NS indicates probability level  $> 0.05$ .

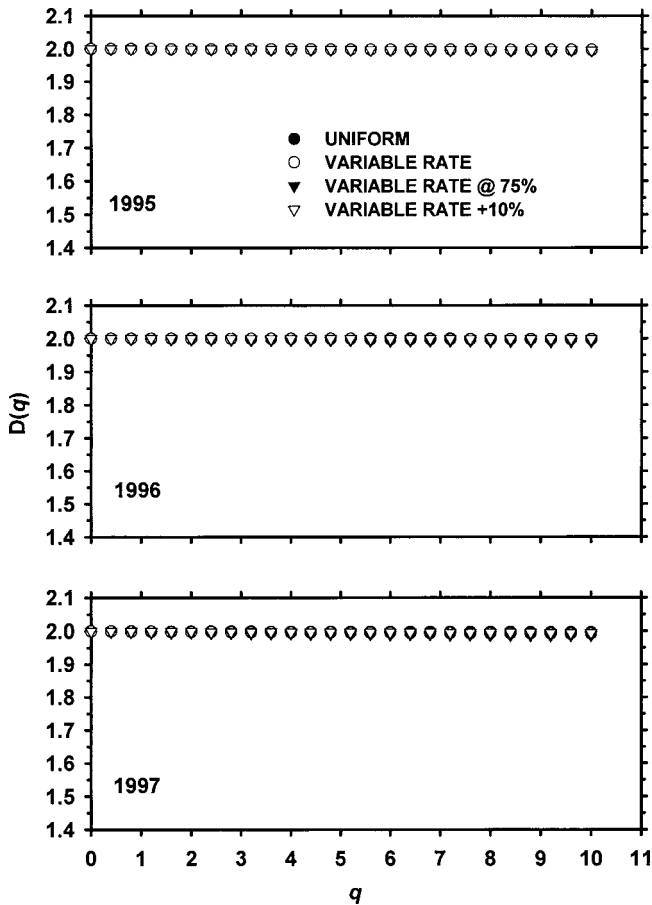


Fig. 5. Corn grain yield  $D(q)$  values as a function of increasing  $q$  values for four N treatments in 3 yr.

patterns of high or low soil  $\text{NO}_3\text{-N}$  areas in the field. Eghball et al. (1997) also observed less spatial variability of corn yield while soil  $\text{NO}_3\text{-N}$  exhibited much stronger spatial variability in another variable rate N study.

The  $D_1$  and  $D_2$  values were all 2 for corn grain yield for all treatments across 3 yr, indicating lack of long-range variation in yield spatial variability. Since long-range variability was unimportant in the corn yield data, small short-range variation became dominant when spatial variability of yield was characterized.

## CONCLUSIONS

Variable rate N application did not result in greater corn grain yield or less spatial variability of residual soil  $\text{NO}_3\text{-N}$  or corn yield than uniform N application. When N application rate was reduced by 25% in the VRAT @ 75% treatment, corn grain yield was basically similar to full rate application but residual soil  $\text{NO}_3\text{-N}$  was significantly reduced, pointing to underestimation of soil N mineralization in the N recommendation equation. Multifractal analysis indicated significantly greater spatial variability for residual soil  $\text{NO}_3\text{-N}$  than corn grain yield. However, no consistent patterns of spatial variability of soil residual  $\text{NO}_3\text{-N}$  and corn grain yield were observed across years. It seems that corn grain yield spatial variability is not significantly influenced by soil

Table 5. The distance between  $a$  values of the treatments for the  $f(a)$  vs.  $a$  lines ( $adiff$ ) for the residual soil nitrate and grain yields (Fig. 3 and 6, respectively) in various years.

N treatment	Soil nitrate				Grain yield		
	1994	1995	1996	1997	1995	1996	1997
Uniform N	0.639	0.462	0.730	0.534	0.02	0.00	0.02
Variable N	0.614	0.524	0.490	0.606	0.02	0.02	0.04
Var. N @ 75%	0.487	0.911	0.506	0.375	0.02	0.05	0.14
Var. N + 10%	0.487	0.957	0.623	0.374	0.02	0.00	0.02
SD	0.081	0.257	0.112	0.117	—	0.02	0.06
PR > T†	0.02	0.01	0.01	0.01	—	NS	NS

† The probability level for soil nitrate tests the hypothesis that the mean difference among treatments is equal to zero while the probability levels for yield test the hypothesis that the actual mean of the treatments is equal to zero; NS indicates probability level > 0.05.

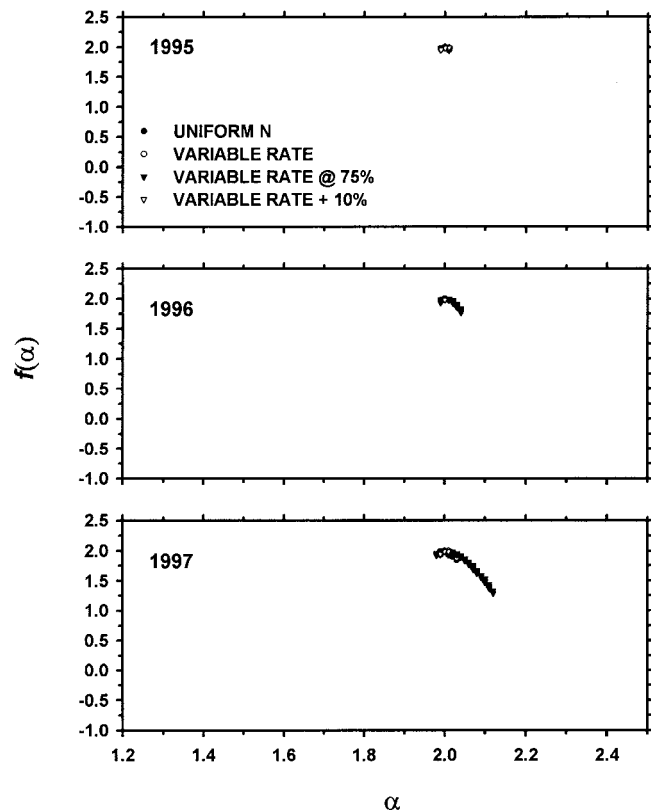


Fig. 6. Multifractal spectrums for corn grain yield for four N treatments in 3 yr.

$\text{NO}_3\text{-N}$  distribution, indicating noneffectiveness of using soil  $\text{NO}_3\text{-N}$  spatial distribution for managing corn yield variability unless some areas in the field are severely N deficient. Increasing the N application rate by 10% over the recommended rate resulted in significant yield reduction, but no difference in residual soil  $\text{NO}_3\text{-N}$  as compared with the recommended rate. Variable rate N application based on spatial variability of soil  $\text{NO}_3\text{-N}$  and organic matter was not more effective in terms of corn grain yield and spatial distribution of residual soil  $\text{NO}_3\text{-N}$  than uniform N application. Multifractal parameters provided quantitative indications of spatial and temporal variability patterns for soil  $\text{NO}_3\text{-N}$  and corn grain yield.



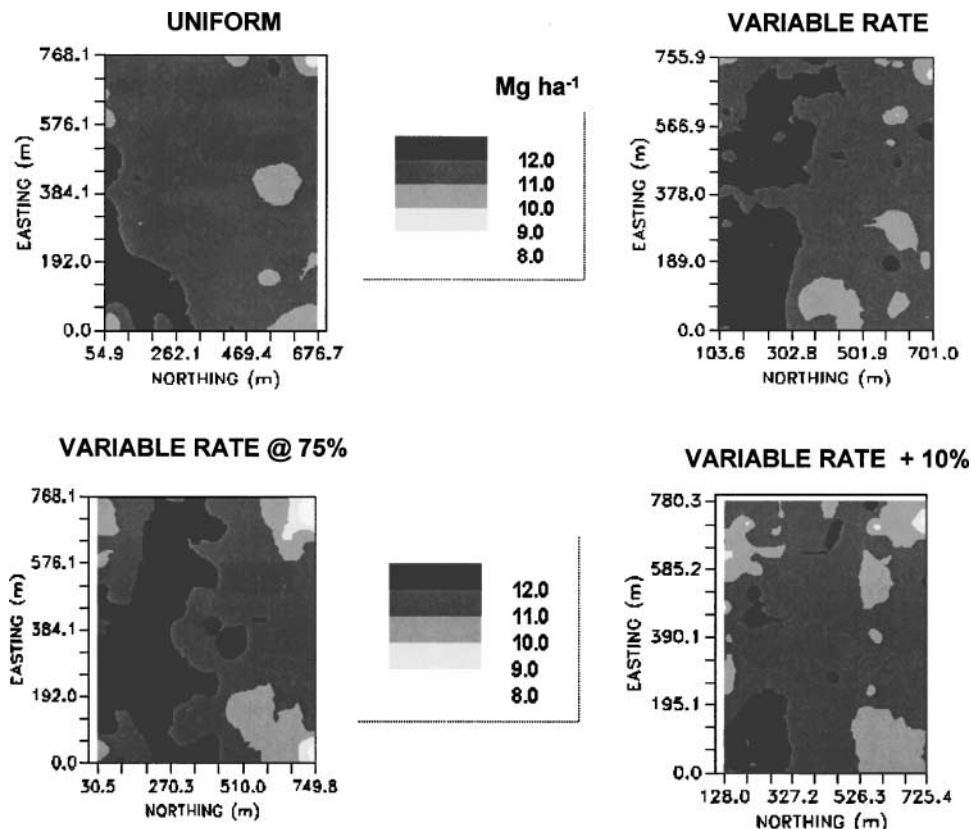


Fig. 7. Corn grain yield distribution for four treatments in 1997. Each map was generated using the data from the strips of that treatment.

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